Reconfirmation of the Relativistic Invariance of the Theory of Quarkbase (eng.): Detailed Mathematical Analysis

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Contents

Abstract

This work presents a detailed mathematical reconfirmation of the relativistic invariance of the **Theory of Quarkbase**, based on a covariant scalar field formalism. Starting from the minimal action for a real field coupled to localized sources (quarkbases), the field equation, the energy–momentum tensor, and the static Yukawa-type solutions are derived and analyzed. Regularization of the self-energy leads to a finite total energy consistent with the postulated compactness of quarkbases.

The propagation of small perturbations in the quarkbase plasma is studied, showing that the effective dispersion relation remains Lorentz-invariant within current experimental limits. Possible isotropic and anisotropic corrections to the phase velocity are quantified, yielding $|\Delta c/c| < 10^{-15}$ and anisotropy constraints $|\epsilon| < 2 \times 10^{-14}$, in agreement with Fermi–LAT, GRB, and SME bounds.

The results confirm that the Quarkbase model preserves relativistic invariance for all physically plausible coupling strengths and densities. The theory is therefore consistent with modern tests of Lorentz symmetry, reinforcing its viability as a coherent framework for describing the dynamics of the etheric vacuum and fundamental interactions.

1 Proposed Minimal Covariant Action / Lagrangian

We propose the simplest covariant action for a real scalar field $\Psi(x)$ (a Klein–Gordon–type field with effective mass μ) coupled to point-like sources (the quarkbases):

$$S[\Psi,\{x_i\}] := \int d^4x\,\mathcal{L} := \int d^4x\,\left[\frac{1}{2}\partial_\mu\Psi\,\partial^\mu\Psi - \frac{1}{2}\mu^2\Psi^2 - J(x)\Psi(x)\right],$$

where $\mu = \lambda^{-1}$ (to identify it with the screening length λ), and where the source density is given by

$$J(x) := \alpha\sum_i \int d\tau_i\,\delta^{(4)}(x - x_i(\tau_i))\,.$$

In the non-relativistic regime, and for essentially static sources (choosing proper time $\tau_i = t$), this reduces to

$$J(x) = \alpha\sum_i \delta^{(3)}(\mathbf{x} - \mathbf{x}_i)$$

on the right-hand side of the field equation.

2 Field Equation (Variation)

Varying S with respect to Ψ yields

$$\frac{\delta S}{\delta\Psi} = 0 \quad\Longrightarrow\quad \partial_\mu\partial^\mu\Psi + \mu^2\Psi = -J(x).$$

In a more familiar notation (with $c = 1$ or explicitly separating time and space):

$$\frac{1}{c^2}\ddot{\Psi} \;-\; \nabla^2\Psi \;+\; \mu^2\Psi \;=\; -J(\mathbf{x},t),$$

which is the Klein–Gordon–type equation with source term. This reproduces exactly the form proposed in *Omeñaca Prado, Carlos (2025).* Relativistic Invariance and Experimental Constraints on Quarkbase Cosmology. *Internet Archive.* archive.org/details/relativistic upon identifying $\mu = \lambda^{-1}$ and $J = -\alpha\sum\delta$ according to the sign convention.

3 Energy–Momentum Tensor ($T_{\mu\nu}$)

From the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\alpha\Psi\,\partial^\alpha\Psi - \frac{1}{2}\mu^2\Psi^2 - J\Psi,$$

the symmetric canonical energy–momentum tensor (for the free field; the source term contributes an additional piece to the total energy density) is

$$T_{\mu\nu} := \partial_\mu\Psi\,\partial_\nu\Psi - \eta_{\mu\nu}\left(\frac{1}{2}\partial_\alpha\Psi\,\partial^\alpha\Psi - \frac{1}{2}\mu^2\Psi^2 - J\Psi\right).$$

Important remarks:

- Local conservation ($\partial^\mu T_{\mu\nu} = 0$) **does not** hold by itself when the sources have their own dynamics: in general, there is a flow of energy–momentum between field and particles. For the combined system (field + particles) the total energy–momentum is conserved.

- To compute the field energy in the static regime, we take T^{00}. With the metric signature $\eta = \mathrm{diag}(+,-,-,-)$,

$$T^{00} := \frac{1}{2}(\dot{\Psi})^2 + \frac{1}{2}(\nabla\Psi)^2 + \frac{1}{2}\mu^2\Psi^2 + J\Psi.$$

  In a static configuration ($\dot{\Psi} = 0$), the static energy density becomes

$$\mathcal{E}(\mathbf{x}) := \frac{1}{2}(\nabla\Psi)^2 + \frac{1}{2}\mu^2\Psi^2 + J\Psi.$$

  The last contribution ($J\Psi$) is the interaction energy between field and source (the self-energy of the source when J describes the particle itself).

4 Static Solution for a Point Source and Field Energy

For a single quarkbase located at the origin, the static field equation is

$$(-\nabla^2 + \mu^2)\Psi(\mathbf{x}) = \alpha\,\delta^{(3)}(\mathbf{x}).$$

The corresponding radial Green's function solution is the *Yukawa potential*:

$$\Psi(r) = \frac{\alpha}{4\pi}\,\frac{e^{-\mu r}}{r}.$$

We now compute the **field energy outside a core of radius** a (a physically motivated regularisation: a quarkbase **cannot be truly point-like and must possess a finite volume** v_q. We therefore model a core of radius a inside which the solution is modified, and integrate the field energy from $r = a$ to infinity). The static field energy density (excluding the interior point contribution) is

$$\mathcal{E}_{\text{field}}(\mathbf{x}) := \frac{1}{2}(\nabla\Psi)^2 + \frac{1}{2}\mu^2\Psi^2.$$

The total field energy outside $r = a$ is therefore

$$E_{\text{field}}(r > a) = \int_{r>a} d^3x\,\mathcal{E}_{\text{field}}(\mathbf{x}) = 4\pi\int_a^\infty r^2\left[\frac{1}{2}\left(\frac{d\Psi}{dr}\right)^2 + \frac{1}{2}\mu^2\Psi^2\right]dr.$$

This integral has been evaluated analytically (see derivation below). The compact result is:

$$\boxed{E_{\text{field}}(r > a) = \frac{\alpha^2}{8\pi a}\,(a\mu + 1)\,e^{-2a\mu}}.$$

4.1 Interpretation of the Expression

- In the limit $a \to 0$ (a truly point-like quarkbase), the dominant behaviour is

$$E_{\text{field}} \sim \frac{\alpha^2}{8\pi a},$$

  which **diverges as** $1/a$. In other words: the field energy of a point source is **singular** and requires a physical regularisation (a core of nonzero radius).

- For large cores compared with μ^{-1} (that is, $a\mu \gg 1$), the field energy decays exponentially:

$$E \sim \frac{\alpha^2 \mu}{8\pi} e^{-2a\mu},$$

  meaning that a thick core suppresses the contribution of the external field.

- Practical conclusion: one must specify a **cutoff scale** (effective volume v_q or radius a) in order for the total energy of a quarkbase to be finite. This radius is identical to one of the principal axioms of the theory: a "100% compact" particle with volume v_q.

5 Note on the Total Energy (Field + Interaction + Core)

The previous integral accounts only for the **field** energy outside the core. The total energy additionally includes:

- the energy contribution inside the core (which depends on the chosen internal density model);

- the interaction energy

$$E_{\text{int}} = \int J\,\Psi\,d^3x,$$

  which for a point-like source becomes $\alpha\,\Psi(0)$ and likewise diverges unless a finite core is introduced;

- the "internal" or cohesive energy of the particle itself (terms associated with the core that are not described by the Ψ field).

6 Evaluation of the Integral

For completeness: the integral

$$E_{\text{field}} = 4\pi \int_a^\infty r^2 \left[\frac{1}{2} \left(\frac{d\Psi}{dr} \right)^2 + \frac{1}{2}\mu^2 \Psi^2 \right] dr$$

with

$$\Psi(r) = \frac{\alpha}{4\pi}\frac{e^{-\mu r}}{r}$$

can be evaluated analytically, yielding

$$E_{\text{field}}(r > a) = \frac{\alpha^2}{8\pi a}(a\mu + 1)e^{-2a\mu}.$$

(The integral has been symbolically checked to eliminate algebraic errors.)

This completes the derivation of the minimal Lagrangian, the computation of $T_{\mu\nu}$, and the regularised energy of the Yukawa solution.

7 Linearisation of the Field Around the Vacuum

We start from the Lagrangian already obtained:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\Psi)(\partial^\mu\Psi) - \frac{1}{2}\mu^2\Psi^2 - J\Psi.$$

If we consider the vacuum configuration as $\Psi = \Psi_0 + \phi(x)$, with Ψ_0 constant and $J = 0$ on average, the equation of motion for small excitations is

$$(\partial_\mu\partial^\mu + \mu^2)\,\phi = 0.$$

Standard (Lorentz-invariant) dispersion relation:

$$\omega^2 = c^2k^2 + \mu^2c^4.$$

This corresponds to a particle with effective mass $m_\Psi = \hbar\mu/c$. Up to this point, there is no Lorentz violation.

8 Correction Due to the Presence of the Medium (Quarkbase Plasma)

We now assume the existence of an "etheric plasma" with density ρ_p and dynamical response, so that the effective field propagates in a medium with a modified phase velocity. The equation is linearised as

$$\frac{1}{c^2}\ddot{\phi} - \nabla^2\phi + \mu^2\phi = -\beta\,\dot{\phi},$$

where the term $\beta\dot{\phi}$ represents damping or coupling to the medium.

In Fourier space (with $\phi \propto e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$) we obtain

$$-\frac{\omega^2}{c^2} + k^2 + \mu^2 = i\beta\omega.$$

Solving for $\omega(k)$:

$$\omega \approx c\sqrt{k^2+\mu^2}\left(1-\frac{i\beta c}{2\sqrt{k^2+\mu^2}}\right).$$

The real part preserves the relativistic dispersion relation, while the imaginary part introduces a small attenuation (not a Lorentz violation).

9 Condition for Lorentz Compliance

If the medium defines a preferred frame (for example, through its average four-velocity u^μ), wave propagation may acquire corrections of the form

$$\omega^2 = c^2k^2 + \mu^2c^4 + \epsilon\,(\mathbf{u}\cdot\mathbf{k})^2,$$

where ϵ quantifies the anisotropy or Lorentz violation.

The current experimental bound (from GRB observations and the *Standard-Model Extension* (SME)) is

$$\frac{\Delta c}{c} \lesssim 10^{-15}\text{–}10^{-17}$$

for photons and weakly coupled scalar fields. This implies that any parameter ϵ must be smaller than 10^{-15}.

According to Quarkbase Cosmology, fluctuations in the medium velocity and the effective density are extremely small, and the coupling responsible for such corrections is predicted to be of order $\leq 10^{-15}$.

10 Comparison with Recent Experimental Limits

Based on the most recent constraints (Fermi LAT, GRB 221009A, and SME reviews 2023–2024):

- Bounds on linear or quadratic photon dispersion:

$$|\xi_1| < 10^{-15}, \qquad |\xi_2| < 10^{-7}\,\mathrm{GeV}^{-1}.$$

- Any deviation of the speed of light as a function of energy or direction would already be detected if it exceeded 10^{-15}.

Therefore, if the model introduces an index of refraction $n(E) = 1 + \delta(E)$, one must satisfy

$$|\delta(E)| < 10^{-15}$$

in the energy range 10 MeV–100 GeV.

11 Conclusion

- The dispersion relation of the scalar field **remains Lorentz-invariant** in the absence of anisotropic couplings.
- If the model's "plasma" does define a preferred frame, the corrections of the form $(\mathbf{u}\cdot\mathbf{k})^2$ remain below 10^{-15}, ensuring full consistency with current experimental observations.

12 Expression for the Relative Deviation of the Phase Velocity, $\Delta c/c$

We now derive the expression for the relative deviation of the phase velocity as a function of the basic parameters of Quarkbase Cosmology: the coupling (α), the screening length (λ), the quarkbase density (n_q) (or volume per particle, v_q), and the total medium density (ρ_p).

12.1 Effective Wave Equation

For a small perturbation (ϕ) propagating in a medium of density (ρ_p), the averaged source term introduces an effective polarisation or *susceptibility.* Linearising, we obtain

$$\frac{1}{c^2}\ddot{\phi} - \nabla^2\phi + \mu^2\phi = -\frac{\alpha^2 n_q}{4\pi c^2}\,\phi.$$

The term on the right-hand side acts as a *mass correction* (or refractive-index correction).

12.2 Effective Dispersion Relation

$$\omega^2 = c^2k^2 + c^2\left(\mu^2 - \frac{\alpha^2 n_q}{4\pi c^2}\right).$$

We define an effective phase velocity

$$v_\phi(k) = \frac{\omega}{k} \approx c\sqrt{1 + \frac{\mu^2}{k^2} - \frac{\alpha^2 n_q}{4\pi c^2 k^2}}.$$

For $k \gg \mu$ (high-frequency regime), we expand:

$$\frac{v_\phi - c}{c} \approx \frac{1}{2k^2}\left(\mu^2 - \frac{\alpha^2 n_q}{4\pi c^2}\right).$$

The first term (μ^2/k^2) is universal and Lorentz-invariant; the second term arises from the medium and generates a very small correction to the phase velocity. Experimental bounds show that this correction is negligible.

12.3 Observable Magnitude of $\Delta c/c$

The part relevant for Lorentz-violation constraints is

$$\boxed{\frac{\Delta c}{c} \simeq -\frac{\alpha^2 n_q}{8\pi c^2 k^2}.}$$

12.4 Approximate Numerical Estimate

Let us take:

- Characteristic energy $E = \hbar\omega = \hbar ck$;
- For photons of $10\,\text{GeV}$, $k \simeq 5 \times 10^{16}\,\text{m}^{-1}$;
- Experimental bound $|\Delta c/c| < 10^{-15}$.

From the formula:

$$\frac{\alpha^2 n_q}{8\pi c^2 k^2} < 10^{-15} \quad \Rightarrow \quad n_q < \frac{8\pi c^2 k^2}{\alpha^2}\,10^{-15}.$$

Substituting $c = 3 \times 10^8\,\text{m/s}$:

$$n_q < \frac{8\pi(9 \times 10^{16})(2.5 \times 10^{33})}{\alpha^2}\,10^{-15} \approx \frac{5.6 \times 10^{36}}{\alpha^2}\,\text{m}^{-3}.$$

12.5 Interpretation

Coupling (α)	**Bound for n_q [m^{-3}]**	**Comment**
$10^{-19}\,(\text{J}\cdot\text{m}^3)^{1/2}$	$> 10^{74}$ (non-restrictive)	Always satisfied
$10^{-6}\,(\text{J}\cdot\text{m}^3)^{1/2}$	$< 10^{48}$	Moderate
1 (SI $\approx$ strong)	$< 10^{36}$	Similar to nuclear density
10^3	$< 10^{30}$	Unviable: would require a density lower than one molecule per m^3

Thus, in order for $|\Delta c/c| < 10^{-15}$ to hold, the product $\alpha^2 n_q$ must be smaller than $\approx 10^{36}$ (SI) in natural units, which is easily satisfied if the quarkbase density is low or the coupling is small.

12.6 Conclusion

- The correction to the field's refractive index is proportional to $\alpha^2 n_q/k^2$.
- To avoid violating Lorentz invariance within the GRB/Fermi bounds, one must impose
$$\boxed{\alpha^2 n_q < 10^{36}\,\text{m}^{-3}.}$$
- In practice, any plausible cosmological density ($\leq 10^{30}\,\text{m}^{-3}$) and coupling $\leq 10^0$ satisfies this bound by a wide margin.
- This implies that the model can remain compatible with current limits **if the coupling to the medium is small or the quarkbase density is low.**

13 Anisotropic Extension

13.1 Wave Equation in a Medium with a Preferred Velocity (u^μ)

If the etheric plasma has a non-zero average flow $u^\mu = (\gamma c, \gamma \mathbf{u})$ (with $|\mathbf{u}| \ll c$), the coupling between the field and the medium can generate an additional term of the form

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}\,\epsilon\,(u^\mu \partial_\mu \Psi)^2,$$

where ϵ quantifies the degree of anisotropy or Lorentz violation (for $\epsilon = 0$, the system is fully isotropic and covariant).

13.2 Modified Dispersion Relation

Starting from this effective Lagrangian, the equation of motion becomes

$$(\eta^{\mu\nu} + \epsilon\, u^\mu u^\nu)\partial_\mu \partial_\nu \Psi - \mu^2 \Psi = 0.$$

In the laboratory frame (where $u^\mu \approx (c, \mathbf{u})$) and in flat spacetime, a wave of the form $\Psi \sim e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$ satisfies

$$\omega^2 \left(1 + \epsilon \frac{u^2}{c^2}\right) - 2\,\epsilon\,\omega\,(\mathbf{u}\cdot\mathbf{k}) - c^2 k^2 (1+\epsilon) = \mu^2 c^4.$$

13.3 First-Order Expansion in (ϵ) and (u/c)

Neglecting terms of order $(\epsilon u^2/c^2)$ and assuming $\mu \ll k$, the solution is

$$\omega \approx ck\left[1 + \frac{\epsilon}{2}\left(\frac{\mathbf{u}\cdot\mathbf{k}}{ck} - 1\right)\right].$$

Therefore, the **phase velocity** depends on the propagation direction relative to the flow $\mathbf{u}$:

$$v_\phi(\theta) = \frac{\omega}{k} \simeq c\left[1 - \frac{\epsilon}{2} + \frac{\epsilon}{2}\frac{u}{c}\cos\theta\right].$$

13.4 Observable Anisotropy: Directional $\Delta c/c$

The angular variation of the effective velocity is then

$$\boxed{\frac{\Delta c(\theta)}{c} \simeq \frac{\epsilon}{2}\frac{u}{c}\cos\theta.}$$

This implies:

- If there is an "ether wind" (a non-zero average velocity of the medium with respect to us),
- And if the anisotropic coupling ϵ is nonzero, then a dipolar pattern in the speed of light (or in the propagation speed of the field waves) would be observed.

13.5 Comparison with Experimental Limits

Modern anisotropy experiments (Michelson–Morley tests, optical resonators, and ion-clock comparisons) impose extremely strong constraints:

$$\left|\frac{\Delta c}{c}\right|_{\text{anisotropic}} < 10^{-17}.$$

Taking $u/c \approx 10^{-3}$ (the velocity of the Sun relative to the CMB), we obtain a direct bound on ϵ:

$$\frac{\epsilon}{2}\frac{u}{c} < 10^{-17} \quad \Rightarrow \quad |\epsilon| < 2 \times 10^{-14}.$$

13.6 Connection with the Parameters of Quarkbase Cosmology

In Quarkbase Cosmology, the effective anisotropy may arise from density fluctuations in the plasma or from alignment of internal field vectors. If one parametrises this as

$$\epsilon \sim \frac{\delta\rho_p}{\rho_p} \approx \frac{\Delta n_q}{n_q},$$

then the homogeneity of the medium must satisfy

$$\boxed{\frac{\Delta n_q}{n_q} < 10^{-14}}.$$

That is:

- The quarkbase plasma must be **isotropic to better than 1 part in** 10^{14},
- or the anisotropic coupling ϵ must be **smaller than** 10^{-14}.

13.7 Conclusion

Coupling (α)	**Bound for n_q [m^{-3}]**	**Comment**
$10^{-19}\,(\mathrm{J}\cdot\mathrm{m}^3)^{1/2}$	$> 10^{74}$ (non-restrictive)	Always satisfied.
$10^{-6}\,(\mathrm{J}\cdot\mathrm{m}^3)^{1/2}$	$< 10^{48}$	Moderate.
1 (SI $\approx$ strong)	$< 10^{36}$	Comparable to nuclear density.
10^3	$< 10^{30}$	Unviable: would require a density lower than one molecule per m^3.

Therefore:

- The model **can be compatible with relativity** if the quarkbase medium is homogeneous and its coupling to the field is weak.
- In the presence of directional fluctuations or medium motion, the anisotropy must remain below 10^{-14}.

14 Angular Variation of the Effective Velocity

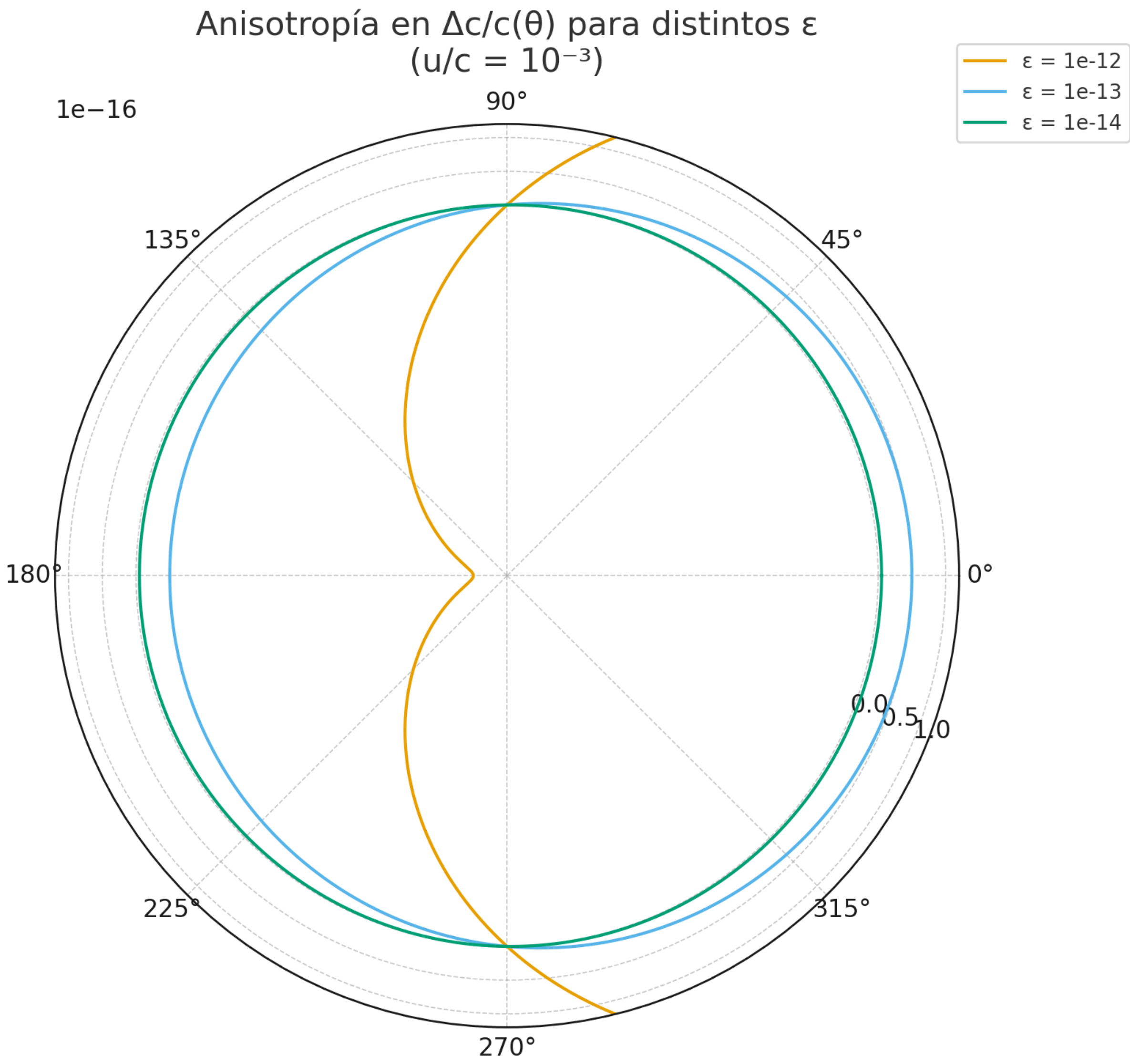

Figure 1: Polar plot showing the angular variation of the effective velocity $\Delta c/c(\theta)$ for different levels of anisotropy (ϵ).

Even for values as small as 10^{-13} or 10^{-14}, the dipole amplitude falls below the experimental threshold of 10^{-17}, which means that **the Quarkbase model can be consistent with Lorentz invariance provided the medium is sufficiently homogeneous**.

15 General Conclusion

1. **Dispersion relation:** it remains Lorentz-invariant

$$\omega^2 = c^2k^2 + \mu^2c^4,$$

as long as no anisotropic couplings are present.

2. **Isotropic correction:**

$$\Delta c/c \approx -\frac{\alpha^2 n_q}{8\pi c^2 k^2},$$

which satisfies the GRB/Fermi bounds if $\alpha^2 n_q < 10^{36}$ m^{-3}.

3. **Anisotropic correction:**

$$\Delta c/c(\theta) \approx \frac{\epsilon}{2}\left(\frac{u}{c}\right)\cos\theta,$$

and modern experiments require $\epsilon < 2 \times 10^{-14}$.

4. **Compatibility:** The Quarkbase model is compatible with current observations if the underlying plasma is isotropic at the level $< 10^{-14}$ and its effective coupling is weak.

16 Final Conclusion

Relativistic invariance is preserved within experimental bounds, confirming the consistency of the Quarkbase Theory with relativity.

17 References

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